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Application of Elzaki Transform to System of Linear Differential Equations

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Abstract

In this paper, The new method using Elzaki transform is presented to solve system of homogeneous and non-homogeneous linear differential equations of first order and first degree with constant coefficients and satisfying some initial conditions. The solution obtained for the system of homogeneous and non-homogeneous linear differential equations of first order and first degree is also discussed. These results prove that the Elzaki transform new method is quite capable, well appropriate to solve such types of problems.

Keywords

Elzaki transform, Inverse Elzaki transform, Circle, System of linear homogeneous and non-homogeneous Differential Equations

I. INTRODUCTION

The differential equations have played a central role in every aspect of applied mathematics, physics and Engineering. Many researchers have paid attention to find the solution of differential equations by using various integral transforms.

Recently, In 2011 Tariq Elzaki introduced a new transform called as Elzaki transform, which is a useful technique for solving ordinary & partial differential equations in the time domain [1],[2],[3],[4]

In this paper, the basic theory of solutions of system of linear differential equations of first order and first degree by Elzaki transform is discussed and it is showed that the Elzaki transform technique is useful to find the solution of system of linear differential equations of first order and first degree with constant coefficient and satisfying some initial conditions.

1.1 Elzaki Transform

A new transform called the Elzaki transform defined for function of exponential order, we consider functions in the set A defined by

$$A = \left\{ f(t) : \exists M, K_1, K_2 > 0, |f(t)| < Me^{\frac{|t|}{k_j}}, if, t \in (-1)^j X[0,\infty) \right\} \qquad (1)$$

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II. METHOD

Consider the non homogeneous system of two linear differential equations of first order and first degree with initial conditions

$$\frac{dx}{dt} = a_1 x + b_1 y + f(t) \tag{3}$$

$$\frac{dy}{dt} = a_2 x + b_2 y + g(t) \tag{4}$$

with initial conditions

$x = c_1$ *and* $y = c_2$, *When* $t = 0$

We assume that the Elzaki transform of

$x(t), y(t), \frac{dx}{dt}, \frac{dy}{dt}, f(t)$ *and* $g(t)$ *exists.*

On taking the Elzaki transform on both sides of equations (3) and (4), we have

$$E\left[\frac{dx}{dt}\right] = a_1 E[x] + b_1 E[y] + E[f(t)] \quad \text{and} \tag{5}$$

$$E\left[\frac{dy}{dt}\right] = a_2 E[x] + b_2 E[y] + E[g(t)] \tag{6}$$

Suppose $E\left[x(t)\right] = T_1(u)$ *and* $E\left[y(t)\right] = T_2(u)$ *then*

$$\left[\frac{T_1(u)}{u} - u.x(0)\right] = a_1 T_1(u) + b_1 T_2(u) + E\{f(t)\} \quad \text{and}$$

$$\left[\frac{T_2(u)}{u} - u.y(0)\right] = a_2 T_1(u) + b_2 T_2(u) + E\{g(t)\}$$

$$\left[\frac{1}{u} - a_1\right] T_1(u) - b_1 T_2(u) = u.c_1 + E\{f(t)\} \quad \text{and} \tag{7}$$

$$\left[\frac{1}{u} - b_2\right] T_2(u) - a_2 T_1(u) = u.c_2 + E\{g(t)\} \tag{8}$$

Solving equations (7) and (8), for $T_1(u)$ and $T_2(u)$, we have

$$T_2(u) = \frac{\left[a_2 c_1 + (\frac{1}{u} - a_1)\right] u + a_2 E\{f(t)\} + \left(\frac{1}{u} - a_1\right) E\{g(t)\}}{\left[\left(\frac{1}{u} - a_1\right)\left(\frac{1}{u} - b_2\right) - a_2 b_1\right]} \tag{09}$$

$$T_1(u) = \frac{\left[b_1 c_2 + (\frac{1}{u} - b_2)\right] u + b_1 E\{g(t)\} + \left(\frac{1}{u} - b_2\right) E\{f(t)\}}{\left[\left(\frac{1}{u} - a_1\right)\left(\frac{1}{u} - b_2\right) - a_2 b_1\right]} \tag{10}$$

By taking the Inverse Elzaki transform on (09) and (10), we get the required solutions.

$x(t) = E^{-1}(T_1(u))$ and $y(t) = E^{-1}(T_2(u))$.

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IV. APPLICATION

In this section, the effectiveness and the usefulness of Elzaki transform technique are demonstrated by finding exact solution of a system of homogeneous and non homogeneous Linear differential equations of first order and first degree with constant coefficients and satisfying some initial conditions.

Example (1) Find the general solution of the system of equations

$$\frac{dx}{dt}+y=2\cos t \qquad (11)$$

$$\frac{dy}{dt}-x=1 \qquad (12)$$

With initial conditions x(0) = -1 and y(0) = 1

Using Elzaki transform on both sides of (11) and (12), we have

$$E\left[\frac{dx}{dt}\right]+E[y]=2E\{\cos t\} \qquad (13)$$

$$E\left[\frac{dy}{dt}\right]-E[x]=E[1] \qquad (14)$$

Suppose $T_1(u)=E[x(t)]$ and $T_2(u)=E[y(t)]$

$$\frac{T_1(u)}{u}-u.x(0)+T_2(u)=2.\frac{u^2}{1+u^2}$$

$$\frac{T_1(u)}{u}+T_2(u)=2.\frac{u^2}{1+u^2}-u \qquad \text{and}$$

$$\frac{T_2(u)}{u}-u.y(0)-T_1(u)=u^2$$

$$\frac{T_2(u)}{u}-T_1(u)=u+u^2$$

$$T_1(u)+u.T_2(u)=2.\frac{u^3}{1+u^2}-u^2 \qquad (15)$$

$$T_2(u)-u.T_1(u)=u^2+u^3 \qquad (16)$$

Solving (15) and (16) , for $T_1(u)$ and $T_2(u)$. Then

Appling Inverse Elzaki transforms,

Thus the required solution of given differential equations are

x(t) = t cos t -1 and y(t) = t.sin t + cos t.

Example (2) Find the general solution of the system of equations

$$\frac{dx}{dt}+\theta y=0 \qquad (17)$$

$$\frac{dy}{dt}-\theta x=0 \qquad (18)$$

With initial conditions $x(0)=c_1$ and $y(0)=c_2$, where c_1 and c_2 are arbitrary constants

Using Elzaki transform on both sides of (19) and (20), we have

$$E\left[\frac{dx}{dt}\right]+\theta E[y]=0$$

$$E\left[\frac{dy}{dt}\right]-\theta E[x]=0$$

Suppose $T_1(u)=E[x(t)]$ *and* $T_2(u)=E[y(t)]$

$$\frac{T_1(u)}{u}-u.x(0)+\theta.T_2(u)=0$$

$$\frac{T_1(u)}{u}+\theta.T_2(u)=u.c_1 \quad and \quad (19)$$

$$\frac{T_2(u)}{u}-u.y(0)-\theta.T_1(u)=0$$

$$\frac{T_2(u)}{u}-\theta.T_1(u)=u.c_2 \quad (20)$$

Solving (19) and (20), we have

$$T_1(u)=c_1\left[\frac{u^2}{1+\theta^2u^2}\right]-c_2.\theta\left[\frac{u^3}{1+\theta^2u^2}\right] \quad and \quad (21)$$

$$T_2(u)=c_1.\theta\left[\frac{u^3}{1+\theta^2u^2}\right]+c_2\left[\frac{u^2}{1+\theta^2u^2}\right] \quad (22)$$

Appling Inverse Elzaki transform on (23) and (24), we get

$x(t)=c_1.\cos\theta t-c_2.\sin\theta t$ *and* $y(t)=c_1.\sin\theta t+c_2.\cos\theta t$

Squaring and adding, geometrically, it is a circle $x^2+y^2=C_1^2+C_2^2$

V. FIGURES AND TABLES

FIGGURE-I

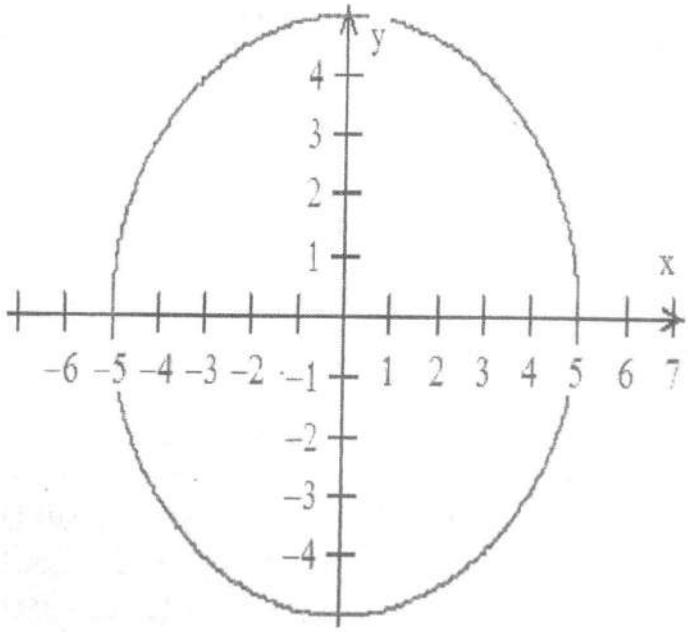

TABLE-I

T	x	y	t	x	y	t	x	y
0.00000	3.00000	4.00000	2.13628	2.34197	-4.41760	4.27257	-4.99432	-0.23816
0.12566	1.91099	4.62040	2.26195	3.36701	-3.69639	4.39823	-4.77819	-1.47271
0.25133	0.70191	4.95049	2.38761	4.18048	-2.74292	4.52389	-4.26183	-2.61473
0.37699	-0.55128	4.96952	2.51327	4.73128	-1.61710	4.64956	-3.47768	-3.59246
0.50265	-1.76983	4.67629	2.63894	4.98479	-0.38968	4.77522	-2.47501	-4.34446
0.62832	-2.87718	4.08924	2.76460	4.92509	0.86223	4.90088	-1.31683	-4.82348
0.75398	-3.80374	3.24524	2.89027	4.55593	2.05997	5.02655	-0.07591	-4.99942
0.87965	-4.49129	2.19734	3.01593	3.90051	3.12826	5.15221	1.16978	-4.86124
1.00531	-4.89665	1.01136	3.14159	3.00000	4.00000	5.27788	2.34197	-4.41760
1.13097	-4.99432	-0.23816	3.26726	1.91099	4.62040	5.40354	3.36701	-3.69639
1.25664	-4.77819	-1.47271	3.39292	0.70191	4.95049	5.52920	4.18048	-2.74292
1.38230	-4.26183	-2.61473	3.51858	-0.55128	4.96952	5.65487	4.73128	-1.61710
1.50796	-3.47768	-3.59246	3.64425	-1.76983	4.67629	5.78053	4.98479	-0.38968
1.63363	-2.47501	-4.34446	3.76991	-2.87718	4.08924	5.90619	4.92509	0.86223
1.75929	-1.31683	-4.82348	3.89557	-3.80374	3.24524	6.03186	4.55593	2.05997
1.88496	-0.07591	-4.99942	4.02124	-4.49129	2.19734	6.15752	3.90051	3.12826
2.01062	1.16978	-4.86124	4.14690	-4.89665	1.01136	6.28319	3.00000	4.00000
0.00000	3.00000	4.00000	2.13628	2.34197	-4.41760	4.27257	-4.99432	-0.23816
0.12566	1.91099	4.62040	2.26195	3.36701	-3.69639	4.39823	-4.77819	-1.47271
0.25133	0.70191	4.95049	2.38761	4.18048	-2.74292	4.52389	-4.26183	-2.61473
0.37699	-0.55128	4.96952	2.51327	4.73128	-1.61710	4.64956	-3.47768	-3.59246
0.50265	-1.76983	4.67629	2.63894	4.98479	-0.38968	4.77522	-2.47501	-4.34446
0.62832	-2.87718	4.08924	2.76460	4.92509	0.86223	4.90088	-1.31683	-4.82348
0.75398	-3.80374	3.24524	2.89027	4.55593	2.05997	5.02655	-0.07591	-4.99942
0.87965	-4.49129	2.19734	3.01593	3.90051	3.12826	5.15221	1.16978	-4.86124
1.00531	-4.89665	1.01136	3.14159	3.00000	4.00000	5.27788	2.34197	-4.41760
1.13097	-4.99432	-0.23816	3.26726	1.91099	4.62040	5.40354	3.36701	-3.69639
1.25664	-4.77819	-1.47271	3.39292	0.70191	4.95049	5.52920	4.18048	-2.74292
1.38230	-4.26183	-2.61473	3.51858	-0.55128	4.96952	5.65487	4.73128	-1.61710
1.50796	-3.47768	-3.59246	3.64425	-1.76983	4.67629	5.78053	4.98479	-0.38968
1.63363	-2.47501	-4.34446	3.76991	-2.87718	4.08924	5.90619	4.92509	0.86223
1.75929	-1.31683	-4.82348	3.89557	-3.80374	3.24524	6.03186	4.55593	2.05997
1.88496	-0.07591	-4.99942	4.02124	-4.49129	2.19734	6.15752	3.90051	3.12826
2.01062	1.16978	-4.86124	4.14690	-4.89665	1.01136	6.28319	3.00000	4.00000

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FIGURE-II

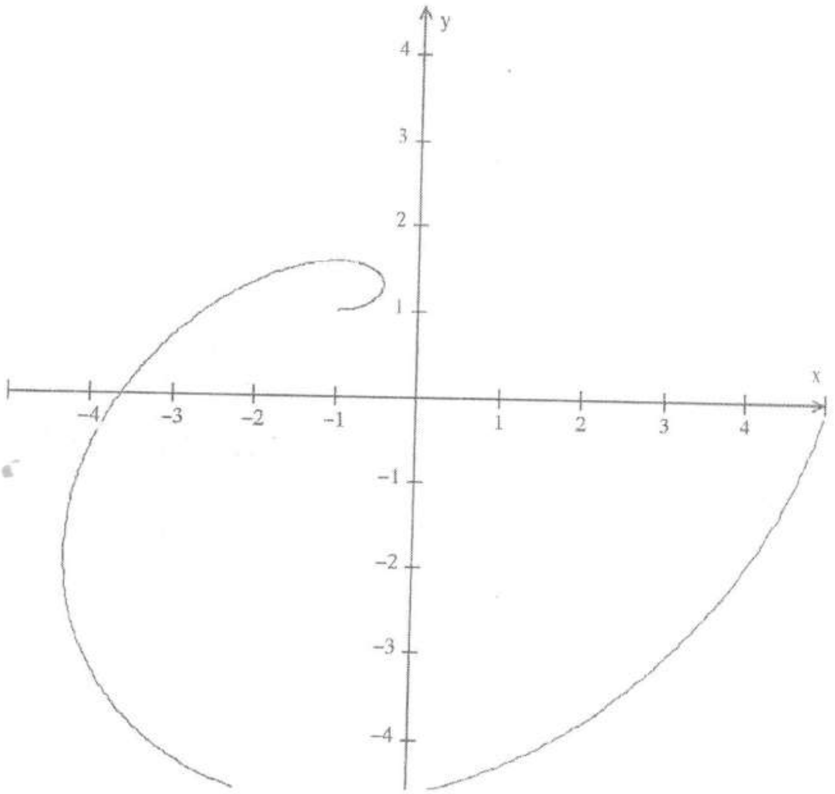

TABLE-II

t	x	y	t	x	y	t	x	y
0.00000	-1.00000	1.00000	2.13628	-2.14468	1.26790	4.27257	-2.81917	-4.29171
0.12566	-0.87533	1.00786	2.26195	-2.44182	1.10544	4.39823	-2.35913	-4.49198
0.25133	-0.75657	1.03109	2.38761	-2.74049	0.90546	4.52389	-1.84769	-4.63114
0.37699	-0.64948	1.06856	2.51327	-3.03328	0.66825	4.64956	-1.29195	-4.70317
0.50265	-0.55952	1.11846	2.63894	-3.31252	0.39501	4.77522	-0.70016	-4.70301
0.62832	-0.49168	1.17833	2.76460	-3.57046	0.08794	4.90088	-0.08167	-4.62670
0.75398	-0.45037	1.24510	2.89027	-3.79946	-0.24980	5.02655	0.55329	-4.47151
0.87965	-0.43929	1.31520	3.01593	-3.99215	-0.61412	5.15221	1.19371	-4.23608
1.00531	-0.46133	1.38464	3.14159	-4.14159	-1.00000	5.27788	1.82803	-3.92043
1.13097	-0.51845	1.44911	3.26726	-4.24149	-1.40161	5.40354	2.44435	-3.52607
1.25664	-0.61168	1.50415	3.39292	-4.28633	-1.81237	5.52920	3.03062	-3.05603
1.38230	-0.74098	1.54520	3.51858	-4.27150	-2.22505	5.65487	3.57488	-2.51483
1.50796	-0.90531	1.56778	3.64425	-4.19348	-2.63194	5.78053	4.06552	-1.90849
1.63363	-1.10258	1.56761	3.76991	-4.04992	-3.02492	5.90619	4.49144	-1.24444
1.75929	-1.32966	1.54075	3.89557	-3.83975	-3.39567	6.03186	4.84236	-0.53148
1.88496	-1.58248	1.48368	4.02124	-3.56323	-3.73584	6.15752	5.10897	0.22037
2.01062	-1.85608	1.39348	4.14690	-3.22202	-4.03717	6.28319	5.28319	1.00000

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In the set A, the constant M is finite number and K_1, K_2 are finite or infinite

The Elzaki transform denoted by the operator E (.) and defined by the Integral Equation.

$$E[f(t)] = T(u) = u.\int_0^\infty f(t)e^{\frac{-t}{u}}\,dt, t \geq 0, K_1 \leq u \leq K_2, 0 \leq t \leq \infty \qquad (2)$$

The variable u in this transform is used to factor the variable t in the argument of the function.

1.2 Inverse Elzaki Transform

We have the problem of finding the Elzaki Transform

$$E[f(t)] = T(u) = u.\int_0^\infty f(t)e^{\frac{-t}{u}}\,dt, t \geq 0, K_1 \leq u \leq K_2, 0 \leq t \leq \infty$$

Of given function f(t),

Now we go in the reverse direction and solve the converse problem, that is given function T(u), then, to find the solution f(t) of which T(u) and is written as $f(t) = E^{-1}(T(u))$, Thus if $E(f(t)) = T(u)$ then $f(t) = E^{-1}(T(u))$

1.3 Elzaki Transform of the Some Functions

We have Elzaki transform of simple functions.

If $f(t)=1$ *then*; $E(1) = u\int_0^\infty e^{\frac{-t}{u}}\,dt = u\left[-ue^{\frac{-t}{u}}\right] = u^2$

If $f(t)=t$ *then*; $E[t] = u\int_0^\infty t.e^{\frac{-t}{u}}\,dt = u^3$

If $f(t)=e^{at}$ *then*; $E(e^{at}) = u.\int_0^\infty e^{at}e^{\frac{-t}{u}}\,dt = \frac{u^2}{1-au}$

If $f(t)=\cos at$, *then*; $E(\cos at) = u.\int_0^\infty \cos at.e^{\frac{-t}{u}}\,dt = \frac{u^2}{1+a^2u^2}$

If $f(t)=\sin at$, *then*; $E(\sin at) = u.\int_0^\infty \sin at.e^{\frac{-t}{u}}\,dt = \frac{au^3}{1+a^2u^2}$

If $f(t)=t\sin at$, *then*; $E(t\sin at) = u.\int_0^\infty t\sin at.e^{\frac{-t}{u}}\,dt = \frac{2au^4}{1+a^2u^2}$

If $f(t)=t\cos at$, *then*; $E(t\cos at) = u.\int_0^\infty t\cos at.e^{\frac{-t}{u}}\,dt = \frac{u(1-a^2u^2)}{(1+a^2u^2)^2}$

Theorem *if* $E[f(t)] = T(u)$ *then* $E\left[\frac{df}{dt}\right] = E\left[f'(t)\right] = \frac{T(u)}{u} - u.f(0)$

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VI. CONCLUSION

In this study Elzaki transform is used. Using this technique the new method is developed to solve system of homogeneous and non-homogeneous differential equations of first order and first degree with constant coefficients and satisfying some initial conditions. The exact solution obtained by the new method is presented. It proves that the new method using Elzaki transform is very useful to find the solution of homogeneous and non-homogeneous linear system of differential equations.

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