# DIRECT, STEADY STATE, THERMOELASTIC PROBLEM OF A THIN RECTANGULAR PLATE

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Abstract:

This paper is concerned with the direct, steady state problem of thermoelasticity. An attempt is made to determine the temperature, displacement and stress functions of a thin rectangular plate occupying the space $D: 0 \leq x \leq a, -b \leq y \leq b$, with stated boundary conditions. The finite Marchi-Fasulo integral transform technique has been used to obtain the solution of the problem.

Keywords: **Direct steady state thermoelastic problem, thin rectangular plate, thermal stresses, finite Marchi-Fasulo integral transform.**

## INTRODUCTION

During recent years, the theory of thermo elasticity has found considerable applications in the solutions of engineering problems. In modern structures, structural components are mainly modelled as plates, as their differential characteristics enable engineers to design better and lighter structures. Hence, the thermo elastic behaviour of rectangular plates is of keen interest in the field of mechanics, civil, aerospace, marine and automobile engineering.

Tanigawa et al. [1] have studied thermal stress analysis of a rectangular plate and its thermal stress intensity factor for compressive stress field. Vihak et al. [2] have investigated the solution of the plane thermo elasticity problem for a rectangular domain. Adam et al. [3] have determined thermo elastic vibration of a laminated rectangular plate subjected to a thermal shock. Ghadle et al. [4] have studied the study of an inverse steady state thermo elastic problem of a thin rectangular plate. Gaikwad et al. [5] have studied the quasi-static thermal stresses in a thick rectangular plate subjected to constant heat supply on extreme edges where as the initial edges are thermally insulated. Deshmukh. et al. [6] have studied thermal stresses in a simply supported plate with thermal bending moments. Gaikwad et al. [7] have studied three dimensional non-homogeneous thermo elastic problem in a thick rectangular plate due to internal heat generation. Thakare et al. [8] studied thermal stresses of a thin rectangular plate with internal moving heat source.

In this article, the direct steady state problem of thermo elasticity of a thin rectangular plate occupying the space $D: 0 \leq x \leq a, -b \leq y \leq b$, with stated boundary conditions is considered. On the edge $x = 0$, the third kind boundary condition is maintained at $h(y)$, which is a known function of $y$. Also, third kind boundary condition is maintained at $F_1(x)$ on the upper surface, and at $F_2(x)$ on the lower surface.

# THE FINITE MARCHI- FASULO INTEGRAL TRANSFORM AND ITS PROPERTY

The Finite Marchi- Fasulo integral transform of $f(z), -h < z < h$ is defined to be

$$\overline{F}(n) = \int_{-h}^{h} f(z) P_n(z) dz \tag{2.1}$$

Then at each point of $(-h, h)$ at which $f(z)$ is continuous,

$$f(z) = \sum_{n=1}^{\infty} \frac{\overline{F}(n)}{\lambda_n} P_n(z) \tag{2.2}$$

where

$$P_n(z) = Q_n \cos(a_n z) - W_n \sin(a_n z)$$

$$Q_n = a_n(\alpha_1 + \alpha_2)\cos(a_n h) + (\beta_1 - \beta_2)\sin(a_n h)$$

$$W_n = (\beta_1 + \beta_2)\cos(a_n h) + (\alpha_2 - \alpha_1) a_n \sin(a_n h)$$

$$\lambda_n = \int_{-h}^{h} P_n^2(z) dz = h[Q_n^2 + W_n^2] + \frac{\sin(2a_n h)}{2a_n}[Q_n^2 - W_n^2]$$

The eigen values $a_n$ are the solutions of the equation

$$[\alpha_1 a_n \cos(a_n h) + \beta_1 \sin(a_n h)] \times [\beta_2 \cos(a_n h) + \alpha_2 a_n \sin(a_n h)]$$
$$= [\alpha_2 a_n \cos(a_n h) - \beta_2 \sin(a_n h)] \times [\beta_1 \cos(a_n h) - \alpha_1 a_n \sin(a_n h)] \tag{2.3}$$

where $\alpha_1$, $\alpha_2$, $\beta_1$ and $\beta_2$ are constants.

The sum in (2.2) must be taken on n corresponding to the positive roots of the equation (2.3)

Moreover, the integral transform (2.1) has the property:

$$\int_{-h}^{h} \frac{\partial^2 f(z)}{\partial z^2} P_n(z) dz = \frac{P_n(h)}{\alpha_1}\left[\beta_1 f(z) + \alpha_1 \frac{\partial f(z)}{\partial z}\right]_{z=h} - \frac{P_n(-h)}{\alpha_2}\left[\beta_2 f(z) + \alpha_2 \frac{\partial f(z)}{\partial z}\right]_{z=-h} - a_n^2 \overline{F}(n)$$

# FORMULATION OF THE PROBLEM: GOVERNING EQUATION

Consider a thin rectangular plate occupying the space $D: 0 \le x \le a; -b \le y \le b;$

The displacement components $u_x$, and $u_y$ in the x and y directions respectively are represented in the integral form as ( Tanigawa, Y and Komatsubara, Y. ; 1997) are

$$u_x = \int_0^a \left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial y^2} - \nu \frac{\partial^2 U}{\partial x^2}\right) + \alpha T\right] dx \tag{3.1}$$

$$u_y = \int_{-b}^{b} \left[\frac{1}{E}\left(\frac{\partial^2 U}{\partial x^2} - \nu \frac{\partial^2 U}{\partial y^2}\right) + \alpha T\right] dy \tag{3.2}$$

where E, $\nu$ and $\alpha$ are the Young's modulus of elasticity, Poisson's ratio and the linear coefficient of thermal expansion of the material of the plate respectively and $U(x, y)$ is the Airy's stress function which satisfy the differential equation :

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 U(x, y) = -\alpha E \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) T(x, y) \tag{3.3}$$

The equation (4.1) is a second order differential equation whose solution is given by

$$T^*(x,n) = A\cosh(a_n x) + B\sinh(a_n x) + \frac{1}{D^2 - a_n^{\,2}}F(x) \tag{4.4}$$

where $A$ and $B$ are arbitrary constants.
Using (4.2) and (4.3) in (4.4), we obtain the values of $A$ and $B$ as,

$$A = \frac{1}{a_n\cosh(a_n a) - \sinh(a_n a)} \times \left\{ \begin{array}{l} g^*(n)a_n - h^*(n)\sinh(a_n a) - a_n\left[\frac{1}{D^2 - a_n^{\,2}}F(x)\right]_{x=a} + \\ \left[\frac{1}{D^2 - a_n^{\,2}}F(x)\right]_{x=0}\sinh(a_n a) + \left\{\frac{d}{dx}\left[\frac{1}{D^2 - a_n^{\,2}}F(x)\right]\right\}_{x=0}\sinh(a_n a) \end{array} \right\}$$

$$B = \frac{1}{a_n\cosh(a_n a) - \sinh(a_n a)} \times \left\{ \begin{array}{l} h^*(n)\cosh(a_n a) - g^*(n) + \left[\frac{1}{D^2 - a_n^{\,2}}F(x)\right]_{x=a} - \\ \left[\frac{1}{D^2 - a_n^{\,2}}F(x)\right]_{x=0}\cosh(a_n a) - \left\{\frac{d}{dx}\left[\frac{1}{D^2 - a_n^{\,2}}F(x)\right]\right\}_{x=0}\cosh(a_n a) \end{array} \right\}$$

Substituting these values of $A$ and $B$ in (4.4),

$$\begin{aligned} T^*(x,n) = {} & \left[h^*(n) - \left[\frac{1}{D^2 - a_n^{\,2}}F(x)\right]_{x=0} - \left\{\frac{d}{dx}\left[\frac{1}{D^2 - a_n^{\,2}}F(x)\right]\right\}_{x=0}\right] \times \left[\frac{\sinh(a_n(x-a))}{a_n\cosh(a_n a) - \sinh(a_n a)}\right] \\ & + \left[g^*(n) - \left[\frac{1}{D^2 - a_n^{\,2}}F(x)\right]_{x=a}\right] \times \left[\frac{a_n\cosh(a_n x) - \sinh(a_n x)}{a_n\cosh(a_n a) - \sinh(a_n a)}\right] + \frac{1}{D^2 - a_n^{\,2}}F(x) \end{aligned} \tag{4.5}$$

Applying inverse finite Marchi- Fasulo integral transform to the equation (4.5), the expression for temperature $T(x,y)$ becomes

$$\begin{aligned} T(x,y) = {} & \sum_{n=1}^{\infty}\left[h^*(n) - \left[\frac{1}{D^2 - a_n^{\,2}}F(x)\right]_{x=0} - \left\{\frac{d}{dx}\left[\frac{1}{D^2 - a_n^{\,2}}F(x)\right]\right\}_{x=0}\right] \times \frac{P_n(y)}{\lambda_n} \times \left[\frac{\sinh(a_n(x-a))}{a_n\cosh(a_n a) - \sinh(a_n a)}\right] \\ & + \sum_{n=1}^{\infty}\left[g^*(n) - \left[\frac{1}{D^2 - a_n^{\,2}}F(x)\right]_{x=a}\right] \times \frac{P_n(y)}{\lambda_n} \times \left[\frac{a_n\cosh(a_n x) - \sinh(a_n x)}{a_n\cosh(a_n a) - \sinh(a_n a)}\right] \\ & + \sum_{n=1}^{\infty}\left[\frac{1}{D^2 - a_n^{\,2}}F(x)\right] \times \frac{P_n(y)}{\lambda_n} \end{aligned} \tag{4.6}$$

The equation (4.1) is a second order differential equation whose solution is given by

$$T^*(x,n) = A\cosh(a_n x) + B\sinh(a_n x) + \frac{1}{D^2 - a_n^{\ 2}}F(x) \tag{4.4}$$

where $A$ and $B$ are arbitrary constants.

Using (4.2) and (4.3) in (4.4), we obtain the values of $A$ and $B$ as,

$$A = \frac{1}{a_n \cosh(a_n a) - \sinh(a_n a)} \times \left\{ \begin{aligned} & g^*(n)a_n - h^*(n)\sinh(a_n a) - a_n\left[\frac{1}{D^2 - a_n^{\ 2}}F(x)\right]_{x=a} + \\ & \left[\frac{1}{D^2 - a_n^{\ 2}}F(x)\right]_{x=0} \sinh(a_n a) + \left\{\frac{d}{dx}\left[\frac{1}{D^2 - a_n^{\ 2}}F(x)\right]\right\}_{x=0} \sinh(a_n a) \end{aligned} \right\}$$

$$B = \frac{1}{a_n \cosh(a_n a) - \sinh(a_n a)} \times \left\{ \begin{aligned} & h^*(n)\cosh(a_n a) - g^*(n) + \left[\frac{1}{D^2 - a_n^{\ 2}}F(x)\right]_{x=a} - \\ & \left[\frac{1}{D^2 - a_n^{\ 2}}F(x)\right]_{x=0} \cosh(a_n a) - \left\{\frac{d}{dx}\left[\frac{1}{D^2 - a_n^{\ 2}}F(x)\right]\right\}_{x=0} \cosh(a_n a) \end{aligned} \right\}$$

Substituting these values of $A$ and $B$ in (4.4),

$$\begin{aligned} T^*(x,n) = & \\ & \left[h^*(n) - \left[\frac{1}{D^2 - a_n^{\ 2}}F(x)\right]_{x=0} - \left\{\frac{d}{dx}\left[\frac{1}{D^2 - a_n^{\ 2}}F(x)\right]\right\}_{x=0}\right] \times \left[\frac{\sinh(a_n(x-a))}{a_n \cosh(a_n a) - \sinh(a_n a)}\right] \\ & + \left[g^*(n) - \left[\frac{1}{D^2 - a_n^{\ 2}}F(x)\right]_{x=a}\right] \times \left[\frac{a_n \cosh(a_n x) - \sinh(a_n x)}{a_n \cosh(a_n a) - \sinh(a_n a)}\right] + \frac{1}{D^2 - a_n^{\ 2}}F(x) \end{aligned} \tag{4.5}$$

Applying inverse finite Marchi- Fasulo integral transform to the equation (4.5), the expression for temperature $T(x,y)$ becomes

$$\begin{aligned} T(x,y) = & \\ & \sum_{n=1}^{\infty}\left[h^*(n) - \left[\frac{1}{D^2 - a_n^{\ 2}}F(x)\right]_{x=0} - \left\{\frac{d}{dx}\left[\frac{1}{D^2 - a_n^{\ 2}}F(x)\right]\right\}_{x=0}\right] \times \frac{P_n(y)}{\lambda_n} \times \left[\frac{\sinh(a_n(x-a))}{a_n \cosh(a_n a) - \sinh(a_n a)}\right] \\ & + \sum_{n=1}^{\infty}\left[g^*(n) - \left[\frac{1}{D^2 - a_n^{\ 2}}F(x)\right]_{x=a}\right] \times \frac{P_n(y)}{\lambda_n} \times \left[\frac{a_n \cosh(a_n x) - \sinh(a_n x)}{a_n \cosh(a_n a) - \sinh(a_n a)}\right] \\ & + \sum_{n=1}^{\infty}\left[\frac{1}{D^2 - a_n^{\ 2}}F(x)\right] \times \frac{P_n(y)}{\lambda_n} \end{aligned} \tag{4.6}$$

where

$$f^*(n) = \int_{-b}^{b} f(y)P_n(y)dy \, , \, h^*(n) = \int_{-b}^{b} h(y)P_n(y)dy \, , \quad \lambda_n = \int_{-b}^{b} P_n^2(y)dy$$

$$P_n(y) = Q_n \cos(a_n y) - W_n \sin(a_n y)$$

$$Q_n = a_n(\alpha_1 + \alpha_2)\cos(a_n b) + (\beta_1 - \beta_2)\sin(a_n b)$$

$$W_n = (\beta_1 + \beta_2)\cos(a_n b) + (\alpha_2 - \alpha_1)a_n \sin(a_n b)$$

Equation (4.6) is the desired solution of the given problem with

$$\beta_1 = \beta_2 = 1 \ \ and \ \alpha_1 = k_1 \, , \alpha_2 = k_2 \, .$$

Substituting the value of T(x,y) from ( 4.6) in ( 3.3) , the expression for Airy's stress function U(x,y) is

$U(x, y) =$

$$-\alpha E \sum_{n=1}^{\infty} \left[ h^*(n) - \left[ \frac{1}{D^2 - a_n^2} F(x) \right]_{x=0} - \left\{ \frac{d}{dx} \left[ \frac{1}{D^2 - a_n^2} F(x) \right] \right\}_{x=0} \right] \times \frac{P_n(y)}{a_n^2 \lambda_n} \times \left[ \frac{\sinh(a_n(x-a))}{a_n \cosh(a_n a) - \sinh(a_n a)} \right]$$

$$-\alpha E \sum_{n=1}^{\infty} \left[ g^*(n) - \left[ \frac{1}{D^2 - a_n^2} F(x) \right]_{x=a} \right] \times \frac{P_n(y)}{a_n^2 \lambda_n} \times \left[ \frac{a_n \cosh(a_n x) - \sinh(a_n x)}{a_n \cosh(a_n a) - \sinh(a_n a)} \right]$$

$$-\alpha E \sum_{n=1}^{\infty} \left[ \frac{1}{D^2 - a_n^2} F(x) \right] \times \frac{P_n(y)}{a_n^2 \lambda_n} \qquad (4.7)$$

## DETERMINATION OF THE THERMOELASTIC DISPLACEMENT

Substituting the value of $U(x,y)$ from (4.7) in (3.1) and (3.2), one obtains the thermoelastic displacement functions $u_x$ and $u_y$ as

$u_x =$

$$\alpha \sum_{n=1}^{\infty} \left\{ \frac{(1+\nu)P_n(y)}{a_n \lambda_n} - \frac{P_n''(y)}{a_n^3 \lambda_n} \right\} \times$$

$$\left\{ \begin{aligned} & \left[ \left[ h^*(n) - \left[ \frac{1}{D^2 - a_n^2} F(x) \right]_{x=0} - \left\{ \frac{d}{dx} \left[ \frac{1}{D^2 - a_n^2} F(x) \right] \right\}_{x=0} \right] \left[ \frac{1 - \cosh(a_n a)}{a_n \cosh(a_n a) - \sinh(a_n a)} \right] \right. \\ & \left. + \left\{ \left[ g^*(n) - \left[ \frac{1}{D^2 - a_n^2} F(x) \right]_{x=a} \right] \left[ \frac{a_n \sinh(a_n a) - \cosh(a_n a) + 1}{a_n \cosh(a_n a) - \sinh(a_n a)} \right] \right\} \right] \end{aligned} \right\}$$

$$+ \alpha \sum_{n=1}^{\infty} \left\{ \frac{P_n(y)}{\lambda_n} - \frac{P_n''(y)}{a_n^2 \lambda_n} \right\} \times \int_0^a \frac{1}{D^2 - a_n^2} F(x)dx + \alpha\nu \sum_{n=1}^{\infty} \left\{ \frac{P_n(y)}{a_n^2 \lambda_n} \right\} \times \int_0^a \frac{d^2}{dx^2} \left[ \frac{1}{D^2 - a_n^2} F(x) \right] dx$$

(5.1)

$$u_y =$$

$$\alpha\nu\sum_{n=1}^{\infty}\left\{\frac{1}{a_n^{\,2}\lambda_n}\int_{-b}^{b}P_n''(y)dy\right\}$$

$$\times\left\{\begin{array}{l}\left[h^*(n)-\left[\frac{1}{D^2-a_n^{\,2}}F(x)\right]_{x=0}-\left\{\frac{d}{dx}\left[\frac{1}{D^2-a_n^{\,2}}F(x)\right]\right\}_{x=0}\right]\left[\frac{\sinh(a_n(x-a))}{a_n\cosh(a_na)-\sinh(a_na)}\right]\\+\left[g^*(n)-\left[\frac{1}{D^2-a_n^{\,2}}F(x)\right]_{x=a}\right]\left[\frac{a_n\cosh(a_nx)-\sinh(a_nx)}{a_n\cosh(a_na)-\sinh(a_na)}\right]\end{array}\right\}$$

$$+\alpha\sum_{n=1}^{\infty}\left\{\frac{1}{\lambda_n}\int_{-b}^{b}\left[P_n(y)+\frac{\nu}{a_n^{\,2}}P_n''(y)\right]dy\right\}\times\left[\frac{1}{D^2-a_n^{\,2}}F(x)\right]$$

$$-\alpha\sum_{n=1}^{\infty}\left\{\frac{1}{a_n^{\,2}\lambda_n}\int_{-b}^{b}P_n(y)dy\right\}\times\frac{d^2}{dx^2}\left[\frac{1}{D^2-a_n^{\,2}}F(x)\right] \tag{5.2}$$

## DETERMINATION OF STRESS FUNCTIONS

Using (4.7) in (3.10), (3.11) and (3.12), the stress functions are obtained as

$$\sigma_{xx} =$$

$$-\alpha E\sum_{n=1}^{\infty}\left[h^*(n)-\left[\frac{1}{D^2-a_n^{\,2}}F(x)\right]_{x=0}-\left\{\frac{d}{dx}\left[\frac{1}{D^2-a_n^{\,2}}F(x)\right]\right\}_{x=0}\right]\frac{P_n''(y)}{a_n^{\,2}\lambda_n}\left[\frac{\sinh(a_n(x-a))}{a_n\cosh(a_na)-\sinh(a_na)}\right]$$

$$-\alpha E\sum_{n=1}^{\infty}\left[g^*(n)-\left[\frac{1}{D^2-a_n^{\,2}}F(x)\right]_{x=a}\right]\frac{P_n''(y)}{a_n^{\,2}\lambda_n}\left[\frac{a_n\cosh(a_nx)-\sinh(a_nx)}{a_n\cosh(a_na)-\sinh(a_na)}\right]$$

$$-\alpha E\sum_{n=1}^{\infty}\left[\frac{1}{D^2-a_n^{\,2}}F(x)\right]\times\frac{P_n''(y)}{a_n^{\,2}\lambda_n} \tag{6.1}$$

$$\sigma_{yy} =$$

$$-\alpha E\sum_{n=1}^{\infty}\left[h^*(n)-\left[\frac{1}{D^2-a_n^{\,2}}F(x)\right]_{x=0}-\left\{\frac{d}{dx}\left[\frac{1}{D^2-a_n^{\,2}}F(x)\right]\right\}_{x=0}\right]\times\frac{P_n(y)}{\lambda_n}\times\left[\frac{\sinh(a_n(x-a))}{a_n\cosh(a_na)-\sinh(a_na)}\right]$$

$$-\alpha E\sum_{n=1}^{\infty}\left[g^*(n)-\left[\frac{1}{D^2-a_n^{\,2}}F(x)\right]_{x=a}\right]\times\frac{P_n(y)}{\lambda_n}\times\left[\frac{a_n\cosh(a_nx)-\sinh(a_nx)}{a_n\cosh(a_na)-\sinh(a_na)}\right]$$

$$-\alpha E\frac{d^2}{dx^2}\sum_{n=1}^{\infty}\left[\frac{1}{D^2-a_n^{\,2}}F(x)\right]\times\frac{P_n(y)}{a_n^{\,2}\lambda_n} \tag{6.2}$$

$\sigma_{xy} =$

$$\alpha E\sum_{n=1}^{\infty}\left[h^{*}(n)-\left[\frac{1}{D^2-a_n^{\,2}}F(x)\right]_{x=0}-\left\{\frac{d}{dx}\left[\frac{1}{D^2-a_n^{\,2}}F(x)\right]\right\}_{x=0}\right]\times\frac{P_n^{'}(y)}{a_n\lambda_n}\times\left[\frac{\cosh(a_n(x-a))}{a_n\cosh(a_n a)-\sinh(a_n a)}\right]$$
$$+\alpha E\sum_{n=1}^{\infty}\left[g^{*}(n)-\left[\frac{1}{D^2-a_n^{\,2}}F(x)\right]_{x=a}\right]\times\frac{P_n^{'}(y)}{a_n\lambda_n}\times\left[\frac{a_n\sinh(a_n x)-\cosh(a_n x)}{a_n\cosh(a_n a)-\sinh(a_n a)}\right]$$
$$+\alpha E\frac{d}{dx}\sum_{n=1}^{\infty}\left[\frac{1}{D^2-a_n^{\,2}}F(x)\right]\times\frac{P_n^{'}(y)}{a_n^{\,2}\lambda_n} \qquad (6.3)$$

## SPECIAL CASE AND NUMERICAL RESULTS

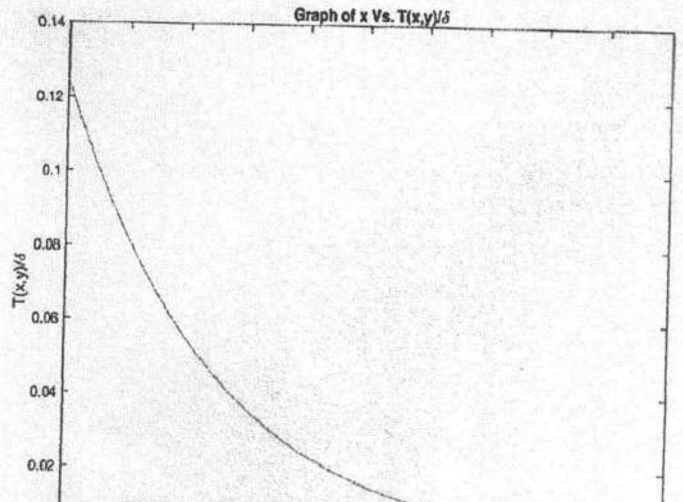
Set $g(y)=(y-b)^2(y+b)^2e^a$ , $h(y)=(y-b)^2(y+b)^2$ , $\delta=16(k_1+k_2), a=2, b=1$ in (4.6), to obtain

$$\frac{T(x,y)}{\delta}=\sum_{n=1}^{\infty}\left[\frac{3a_n\cos^2(a_n)+(a_n^{\,2}-3)\cos(a_n)\sin(a_n)}{a_n^{\,4}\lambda_n}\right]P_n(y)$$
$$\times\left\{\left[\frac{\sinh(a_n(x-2))}{a_n\cosh(2a_n)-\sinh(2a_n)}\right]e^2-\left[\frac{a_n\cosh(a_n x)-\sinh(a_n x)}{a_n\cosh(2a_n)-\sinh(2a_n)}\right]\right\} \qquad (7.1)$$

## CONCLUSION

The temperature, displacement functions and thermal stresses have been determined of a thin rectangular plate, with the stated boundary conditions. The finite Marchi-Fasulo integral transform technique has been used to obtain the numerical results.

The expression (7.1) is represented graphically. It is found that as the value of $x$ increases, the temperature goes on decreasing. Any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the expression. The temperature, displacement and thermal stresses that are obtained can be applied to the design of useful structures or machines in many engineering applications.



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